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MIS6: (Continued)

Case 4: Incomplete Grouped Data

In this case, Kaplan-Meier implies that, unless stated otherwise, assume all deaths (D) occur MOY and split withdrawals (W) and new entrants (E) 50/50 at BOY/EOY

Then use $S_X(t) = \prod_{y_j \le t} \left(1 - \frac{s_j}{r_j}\right)$

Example: (See next page)

M1S6 Example: (Incomplete Grouped Data)

In a mortality study of a cohort of 50 dragons, all age 30, you are given:

(i) Between ages 30 and 31, 10 die, 20 fly away, and 10 enter the study
(ii) Between ages 31 and 32, 15 die, 15 fly away, and 10 enter the study

Determine the Kaplan-Meier estimate for ${}_2q_{30}$.

Age 30 | 31 | 32

30: 50 (start), 10 W, 5 E

31: 10W, 5E | 7.5W, 5E

32: 7.5W, 5E

10 D → $S_1 = 10$

15D → $S_2 = 15$

r_1 = # of dragons at risk of being observed by us to die

$= 50 - 10 + 5 = 45$

Likewise, $r_2 = 45 - 10D - 10W + 5E - 7.5W + 5E = 27.5$

$\therefore$ K-M $\Rightarrow$ ${}_2\hat{p}_{30} = \left(1 - \frac{10}{45}\right)\left(1 - \frac{15}{27.5}\right)$

$\therefore$ ${}_2\hat{q}_{30} = 1 - \frac{35}{45} \cdot \frac{12.5}{27.5}$

MIS7: General Statuses

<u>Recall</u>: T_x = rvr time until the death of (x)

We can think of the life of (x) as a status, $\sigma = x$

If (x) is alive, we say the status σ is intact

~~If~~ When (x) dies, we say the status σ fails

Generalize to other statuses involving multiple lives.

<u>Examples</u>: Assume two lives (x) and (y)

1) Joint-Life Status $\quad \sigma = xy \quad$ (e.g. $\sigma = 40{:}50$)

$\sigma = xy$ fails at the earlier of the deaths of (x) & (y)

Define T_σ = rvr time until the failure of the status $\sigma = xy$

$\therefore \quad T_{xy}$ = rvr time until the first death

<u>Remark</u>: $T_{xy} = \text{Min}(T_x, T_y)$

2) Last-Survivor Status $\quad \sigma = \overline{xy} \quad$ (e.g. $\sigma = \overline{40{:}50}$)

$\sigma = \overline{xy}$ fails at the time of the last death

$T_{\overline{xy}}$ = rvr the time until the last death

<u>Remark</u>: $T_{\overline{xy}} = \text{Max}(T_x, T_y)$

Properties (and Notation) of T_σ

(cdf) $F_{T_\sigma}(t) = F_\sigma(t) = Pr(T_\sigma \le t) = {}_tq_\sigma$

(sf) $S_{T_\sigma}(t) = S_\sigma(t) = Pr(T_\sigma > t) = {}_tp_\sigma$

(pdf) $f_\sigma(t) = {}_t\dot{q}_\sigma = -{}_t\dot{p}_\sigma$

(fom) $\mu_\sigma(t) = \dfrac{f_\sigma(t)}{{}_tp_\sigma} = -\dfrac{d}{dt}\left[\ln({}_tp_\sigma)\right]$

$$\therefore\ {}_np_\sigma = e^{-\int_0^n \mu_\sigma(t)\,dt}$$

<u>Remark</u>: $T_x + T_y = T_{xy} + T_{\overline{xy}}$

$$\Longrightarrow (\ )_x + (\ )_y = (\ )_{xy} + (\ )_{\overline{xy}}$$

E.g.

$${}_tp_x + {}_tp_y = {}_tp_{xy} + {}_tp_{\overline{xy}}$$

$${}_tq_x + {}_tq_y = {}_tq_{xy} + {}_tq_{\overline{xy}}$$

$$f_x(t) + f_y(t) = f_{xy}(t) + f_{\overline{xy}}(t)$$

$$\mathring{e}_x + \mathring{e}_y = \mathring{e}_{xy} + \mathring{e}_{\overline{xy}}$$

$$K_x + K_y = K_{xy} + K_{\overline{xy}}$$

$$e_x + e_y = e_{xy} + e_{\overline{xy}}$$

Exception: (μ's)

$$\mu_x(t) + \mu_y(t) \neq \mu_{xy}(t) + \mu_{\overline{xy}}(t)$$

Reason: $\mu_{\overline{xy}}(t) = \dfrac{f_{\overline{xy}}(t)}{{}_tp_{\overline{xy}}} = \dfrac{f_x(t)+f_y(t)-f_{xy}(t)}{{}_tp_x + {}_tp_y - {}_tp_{xy}}$

$$\frac{a+b-c}{d+e-f} \neq \frac{a}{d} + \frac{b}{e} - \frac{c}{f}$$

Special Case of J-L & L-S Statuses:

$(y) = \overline{n|}$ $\Leftrightarrow$ $\overline{n|}$ is a status that remains intact for exactly n years

I.e. $T_{\overline{n|}} = n$ (not random)

J-L Status: $\sigma = x:\overline{n|}$

$$T_{x:\overline{n|}} = \text{Min}(T_x, n)$$

$\sigma = x:\overline{n|}$ fails at the earlier of the death of (x) and time $t = n$

L-S Status: $\sigma = \overline{x:\overline{n|}}$

$$T_{\overline{x:\overline{n|}}} = \max(T_x, n)$$

$\sigma = \overline{x:\overline{n|}}$ fails at the later of the death of (x) and time $t = n$